Kuwait University | Math. 101 | January 8, 1997
Dept. Math. & Comp. Sci. | Final Examination | Duration: 2 hours

Calculators, Mobile Telephones and Pagers ARE NOT ALLOWED

1 State each of the following(2 points each):

(a) The Intermediate Value Theorem.

(b) Rolle s Theorem.

(c) The Fundamental Theorem of Calculus.

2 Answer each of the following as true or false. Justify your answers(2 points each)

(a) If $\int_a^b f(x)\,dx$ exists then f is differentiable on $[a, b]$

(b) If f is differentiable on $[a, b]$ then $\int_a^b f(x)\,dx$ exists

Each item of each of the following questions is worth 4 points.

3. (a) Evaluate

$$\lim_{x\to\infty} \frac{x^3}{x^2+\sqrt{x^4+x+1}} \sin\frac{1}{x}.$$

b) Let h be differentiable on $(-\infty, \infty)$ and $h(0) = 2$ Find the constant A so that

$$f(x) = \begin{cases} \dfrac{h(x)-h(0)}{\sqrt{x+1}-1} & -1 \le x < 0 \\ A & x \ge 0 \end{cases}$$

is continuous at $x = 0$

4. (a) If $y = \sqrt{u^2+u+7}$ and $u = x^2 + \dfrac{1}{\sqrt[3]{1+\tan x}}$ then find $\dfrac{dy}{dx}$ at $x = 0$

(b) Find the dimensions of the rectangle of maximum area whose perimeter is 100 ft.

5. (a) Find the area of the region bounded by the graphs of $y^2+x-3 = 0$ and $y-x+1 = 0$

(b) The region bounded by the graphs of $y - x^2 - 3 = 0$, $y - 3x + 1 = 0$, $x = 0$ and $x = 1$ is revolved about the line $x = -1$. Find the volume of the resulting solid

6. Evaluate each of the following integrals:

$$(a)\ \int \frac{dw}{\sqrt[3]{(7-5w)^2}} \qquad (b)\ \int_0^{\frac{\pi}{4}} \frac{(1+\tan x)^3 \sec x}{\cos x}\,dx$$

7. (a) Let $f(x) = \frac{1}{\pi}\sqrt{1-x^2}$ Find c such that $f(c)$ is the average value of f on $[0, 1]$

(b) Let $f(x) = \int_1^x \sqrt{1+t^2}\,dt$. Show that $f(x) = 0$ has exactly one solution in $(-\infty, \infty)$